I Congresso do IMECC
7-11/Julho, 2025

ST09 - Fluids, PDEs and Numerical Analysis: Modeling, Methods and Applications

Organizers: Eduardo Abreu (IMECC - UNICAMP), Roberto Ribeiro-Jr (DMAT - UFPR) e Yuri Dumaresq Sobral (MAT - UnB)

Summary: This Thematic Session aims to bring together young and senior researchers working on different aspects of the field Fluids, PDEs and Numerical Analysis to discuss recent and new trends in the field. We would like to bring together experts in Fluids and Numerical Analysis whose work touches on various aspects dynamics of evolutionary PDEs, such as the theory of mathematical modelling of fluid mechanics, wave propagation in fluids, flows in porous media, transport problems, non-Newtonian fluids, and the motion of bodies in viscous fluids from the perspective of nonlinear PDEs, modeling, methods and its applications. Another goal of the special session is to promote idea exchange as well as potential future collaborations and also to provide a space for people from different areas of academia at different stages of their careers, from students to senior researchers that investigate fundamental questions and applications in Fluids, PDEs and Numerical Analysis and related areas, to share new and original concepts as well as to explore new trends and directions emerging in the field.

CRONOGRAMA DAS APRESENTAÇÕES - Quinta, Tarde, 10/Julho/2025:

*** Primeiro Bloco => Quinta (10/Jul), 13:10-14:30: ST09 - (1h20min) – com 20 minutos (15 apresentação + 5 minutos de perguntas)**

1) Bianca M. R. Calsavara (IMECC-Unicamp) - Uniform exponential stability for a thermo-viscoelastic Timoshenko system
2) Erika Paola Ortiz Bernal (IMPA) - Spontaneous Stochasticity in the Fluctuating Navier–Stokes Equations on a Logarithmic Lattice
3) Elias Alfredo Gudiño (UFPR) - Controlled drug release enhanced by temperature
4) Yuri Dumaresq Sobral (UnB) - Immersed granular collapses: a brief overview of recent results

*** Segundo Bloco => Quinta (10/Jul), 16:40-18:20: ST09 - (1h40min) – com 20 minutos (15 apresentação + 5 minutos de perguntas)**

1) Wanderson Lambert (UNIFAL-Poços de Caldas) - The Semi-Discrete Lagrangian-Eulerian Method for Nonlocal Conservation Laws in Several Dimensions
2) André von Borries Lopes (UnB) - Eigenfunction superposition method applied to Poiseuille flows in non-circular ducts
3) Giuseppe Romanazzi (IMECC-Unicamp) - Numerical analysis of a Nonlinear Pressure-Cell density dynamics problem with Non-smooth solutions
4) Ivan R. de Siqueira (PUC-Rio) - Mixed finite element/elliptic mesh generation method for free surface flows: Flow dynamics and operating windows of slot coating applications of thin films **5)** Nikolai Chementov (DCM-FFCLRP/USP) - A boundary control problem for stochastic 2D-Navier-Stokes equations

Abstracts

Eigenfunction superposition method applied to Poiseuille flows in non-circular ducts

André von Borries Lopes
UnB

We present a unified perspective on the application of the eigenfunction superposition method to canonical problems in fluid mechanics, focusing on steady and unsteady Poiseuille flows in ducts of non-circular cross-sections. Three distinct geometries are addressed: triangular ducts (hemi-equilateral and right-angled isosceles), labrys-shaped ducts bounded by confocal ellipses and hyperbolas, and ducts with parabolic segment or lens-shaped cross-sections. For each case, we derive new exact analytical solutions to the governing Poisson or Helmholtz equations, using tailored eigenfunction expansions in coordinate systems adapted to the geometry. These results illustrate the versatility and mathematical elegance of eigenfunction-based techniques, while providing precise benchmarks for validating numerical models in complex domains.

Uniform exponential stability for a thermo-viscoelastic Timoshenko system

Bianca M. R. Calsavara
Unicamp-IMECC

In this work it is established the characterization of the uniform exponential stability for the thermo-viscoelastic Timoshenko beam system under the Fourier law for heat conduction and memory in a history setting given by

$$\begin{cases} \rho_1\phi_{tt} - \kappa(\phi_x + \psi)_x + \sigma\theta_x = 0 & \text{in } (0,L)\times\mathbb{R}^+, \\ \rho_2\psi_{tt} - b\psi_{xx} + \kappa(\phi_x+\psi) + \displaystyle\int_0^\infty g(s)\psi_{xx}(s)\,ds - \sigma\theta = 0 & \text{in } (0,L)\times\mathbb{R}^+, \\ \rho_3\theta_t - \beta\theta_{xx} + \sigma(\phi_x+\psi)_t = 0 & \text{in } (0,L)\times\mathbb{R}^+, \end{cases} \tag{1}$$

subject to initial-boundary conditions

$$\begin{cases} \phi_x(0,t) = \phi_x(L,t) = \psi(0,t) = \psi(L,t) = \theta(0,t) = \theta(L,t) = 0, \quad t \geq 0, \\ (\phi(x,0), \phi_t(x,0), \psi_t(x,0), \theta(x,0)) = (\phi_0, \phi_1, \psi_1, \theta_0), \quad x \in (0,L), \\ \psi(x,t) = \psi_0(x,t), \quad (x,t) \in (0,L)\times(-\infty,0]. \end{cases} \tag{2}$$

Here, the unknown functions $\phi = \phi(x,t), \psi = \psi(x,t)$, and $\theta = \theta(x,t)$ represent, respectively, the vertical displacement, the rotation angle, and the temperature deviation of a beam with length $L > 0$.

Motivated by [1, 2], we explore the intrinsic non differentiable assumption on the memory kernel that provides a necessary and sufficient condition $g(\tau+$

$s) \leq e^{-\delta\tau} g(s), \quad s > 0, \ \tau \geq 0$ for the exponential stability of the whole system. It gives a substantial generalization of the stability results obtained in [3, 4, 5] and surely ties up loose ends about the hypothesis equivalent to exponential stability of the problem. Work done in collaboration with Eduardo H. G. Tavares and Márcio A. Jorge Silva.

References

[1] V.V. Chepyzhov, V. Pata, *Some remarks on stability of semigroups arising from linear viscoelasticity*. Asymptot. Anal. 46 (2006), no. 3-4, pp. 251-273.

[2] M. Conti, F. Dell'oro, V. Pata, *Timoshenko systems with fading memory*. Dynamics of Partial Differential Equations 10.4 (2013), pp. 367-377.

[3] B. Feng, *Uniform decay of energy for a porous thermoelasticity system with past history*. Applicable Anal. 97 (2018), no. 2, pp. 210-229.

[4] M.A. Jorge Silva, R. Racke, *Effects of history and heat models on the stability of thermoelastic Timoshenko systems*. Journal of Differential Equations 275 (2021), pp. 167-203.

[5] A. Soufyane, *Energy decay for porous-thermo-elasticity systems of memory type*, Applicable Anal. 87 (2008), no. 4, pp. 451-464.

Controlled drug release enhanced by temperature

Elias Alfredo Gudiño
UFPR

In this lecture, we will present a new mathematical model to describe the entry of a fluid into a polymeric matrix loaded with drug molecules, followed by its subsequent desorption. The solution of the model is approximated using a semi-implicit finite difference method. Numerical simulations illustrate how the effects of coupling, viscoelastic properties, and temperature can play a central role in the design of controlled drug release devices.

Spontaneous Stochasticity in the Fluctuating Navier–Stokes Equations on a Logarithmic Lattice

Erika Paola Ortiz Bernal
IMPA

We implement numerically a model representing the 3D incompressible Navier-Stokes system with small-scale white noise on logarithmic lattices, i.e., 3D space lattices with logarithmically spaced nodes. Our goal is the numerical analysis of spontaneous stochasticity in this system. For this, we consider decreasing sequences of viscosities and noise parameters and analyze the convergence of the corresponding probability densities, thereby, verifying numerically weak convergence in the inviscid and zero-noise limit. We will report the numerical results obtained in this direction. This is a joint work with Ciro S. Campolina and Alexei A. Mailybaev.

Numerical analysis of a Nonlinear Pressure-Cell density dynamics problem with Non-smooth solutions

Giuseppe Romanazzi
Unicamp-IMECC

I focus on the numerical study of a system of partial differential equations in a nonuniform grid that can be used to describe the non-smooth pressure-cell density dynamics in the colon epithelium. This system couples an elliptic equation with a nonlinear parabolic equation with mixed boundary conditions. It is analyzed as a supra-convergent method in nonuniform grid that guarantees a second order approximation for the system solution gradient using differential operators that have only a first order truncation error. The challenges we face are the construction of the right discretizations of the equations and on the numerical analysis involved considering also the solution non-smoothness.

Mixed finite element/elliptic mesh generation method for free surface flows: Flow dynamics and operating windows of slot coating applications of thin films

Ivan R. de Siqueira
PUC-Rio

Slot coating stands out as one of the most attractive liquid application methods for the scalable production of high-precision, high-performance, multifunctional films, examples of which span from long-standing adhesive tapes and cardboard papers to next-generation photovoltaic modules and battery electrodes. As such, genuinely modeling and fundamentally understanding slot coating applications is critical for optimizing existing manufacturing standards and driving the development of new technologies across a wide range of fields. Here, we review a well-established finite element/elliptic mesh generation method for free surface flows and discuss representative results for the flow dynamics and operating windows of slot coating applications of thin films. Briefly, the equations of motion for Newtonian liquids and mesh generation for the free surface are discretized with Galerkin's weighting functions coupled discontinuous basis functions for pressure and Lagrangian biquadratic basis functions for velocity and mesh position; the resulting system of nonlinear algebraic equations is solved fully implicitly with Newton's method coupled with a frontal LU solver for sparse matrices and a first-order pseudo-arc-length continuation scheme to trace solution branches in the parameter space. We show that the method can effectively capture key features of slot coating flows, including the shape of the free surface in the film formation region, the formation of recirculation regions in the coating bead, and the so-called low-flow limit for the delivery of thin films at high speeds.

A boundary control problem for stochastic 2D-Navier-Stokes equations

Nikolai Chementov
DCM-FFCLRP/USP

In this talk, we discuss a stochastic velocity tracking problem for the 2D-Navier-Stokes equations perturbed by a multiplicative Gaussian noise. From physical point of view, the control acts through a boundary injection/suction device with uncertainty, modelled by non-homogeneous Navier-slip boundary conditions. We show the existence and uniqueness of solution to the state equation and prove the existence of an optimal solution to the control problem. In addition, the first-order necessary optimization conditions are analysed.

N.V. Chemetov acknowledges support from FAPESP, Grant 2024/16483-5: "Theoretical study of mathematical models in fluid dynamics".

Joint work with Fernanda Cipriano (New University of Lisbon, Portugal).

References

[1] N.V. Chemetov, F. Cipriano, *Optimal control of Newtonian fluids in a stochastic environment.* SIAM Journal on Mathematical Analysis, 57 (1), 364-403, 2025.

[2] N.V. Chemetov, F. Cipriano, *A boundary control problem for stochastic 2D-Navier–Stokes equations.* Journal of Optimization Theory and Applications 203 (2), 1847-1879, 2024.

The Semi-Discrete Lagrangian-Eulerian Method for Nonlocal Conservation Laws in Several Dimensions

Wanderson Lambert
UNIFAL-Poços de Caldas

In this talk, we will present the semi-discrete Lagrangian-Eulerian method to approximate a class of multi-dimensional scalar conservation laws with nonlocal flux, referred to as the nonlocal model:

$$\partial_t \rho(t, \mathbf{x}) + \sum_{i=1}^{d} \partial_{x_i} \Big(V^i \Big[W[\rho, \omega](t, \mathbf{x}) \Big] F^i(\rho(t, \mathbf{x})) \Big) = 0, \quad (t, \mathbf{x}) \in (0, T) \times \mathbb{R}^d.$$

This model arises in several important applications, such as traffic flow, porous media, and biological systems, where nonlocal interactions significantly influence the dynamics. The nonlocal term $W[\rho, \omega](t, \mathbf{x})$ introduces spatial averaging, which adds complexity to both the analysis and numerical approximation.

We will analyze the convergence of this method using the weak asymptotic approach, previously developed in [1], and extend its results to the multidimensional nonlocal case. This technique is particularly well-suited for the Lagrangian-Eulerian schemes under consideration. The weak asymptotic method constructs a family of approximate solutions with the following key properties: (1) the approximate functions are uniformly bounded in $L^1(\mathbb{R}^d) \cap L^\infty(\mathbb{R}^d)$; (2) the family is governed by a suitable modulus of continuity in both time and space. These properties allow us to apply an L^1-compactness argument, from which we can extract a convergent subsequence whose limit is a weak entropy solution of the original nonlocal conservation law. To complement the theoretical findings, we will present several numerical examples. This talk presents results from [2] and was conducted in collaboration with E. Abreu, R. de la Cruz, and J. Juajibioy.

References

[1] E. Abreu, M. Colombeau, E. Panov, Weak asymptotic methods for scalar equations and systems, *J. Math. Anal. Appl* 444 (2016) 1203-1232.

[2] E. Abreu, R. De la Cruz, J. Juajibioy, W. Lambert, Semi-discrete Lagrangian-Eulerian approach based on the weak asymptotic method for nonlocal conservation laws in several dimensions, Journal of Computational and Applied Mathematics, Volume 458, April (2025) 116325.

Immersed granular collapses: a brief overview of recent results

Yuri Dumaresq Sobral
UnB

The collapse of granular columns in a viscous fluid is a common model case for submarine geophysical particulate flows, in particular of submarine landslides. In this presentation, we will discuss how three parameters of this problem can significantly affect the dynamics of flow, especially the (normalised) runout distance. Firstly, the influence of the aspect ratio of the columns will be investigated via numerical simulations using a CFD-DEM approach. Three different regimes (free-fall, inertial and viscous) for the collapse will be identified and characterised. The influence of the shedded fluid eddies during the collapse in the inertial regime will be detailed. Secondly, the initial packing density of the column and its influence on the runout distance will be investigated using a LBM-DEM approach. We will discuss how dense packings result in slow dynamics and short runout distances, while loose packings are associated with fast dynamics and long runout distances. We will also discuss how hydroplaning can take place in these flows and enhance their runout distance. Finally, we will discuss the effects of the column size on immersed granular collapses. In this case, laboratory-scale experiments of underwater granular collapses with three different column sizes are carried out, together with their numerical simulations using the coupled LBM-DEM method. We observe non-trivial unscaled behaviours that indicate that as the column size increases, there is more potential energy being transferred into fluid and particle kinetic energies, with increased efficiency as the size of the column increases, so that larger normalised runout distances are observed in larger cases. Joint work with Gengchao Yang, Fiona C. Y. Kowk and Lu Jing.